

# **Business Energy Professional – Math Supplement**

As a supplement to the Business Energy Professional course, we routinely hear from seminar or webinar attendees that they wish there was an overview of math skills needed for the course. This handout contains an overview of the math skills needed for simplifying expressions, converting units, utilizing calculators, and solving equations typically found in the Business Energy Professional Course. Please recognize that anything you do prior to the course in reviewing this material will substantially increase what you learn during the seminar or webinar and will also service those very well as they prepare for the certification exam. This handout is broken into the following sections:

## **Order of Operations / Hierarchy**

## **Calculators**

## **Expressions**

## **Conversion Factors for Unit Conversions**

## **Unit Conversions**

## **Equations**

While this handout is a short refresher of several mathematical operations, the Association of Energy Engineers does offer a very extensive pre-course titled “The Basics of Energy Management”. This online self study course is not required for the Business Energy Professional course or its certification. Some individuals may find the online course worth their while as a review course. The Basics of Energy Management self study seminar outline is as follows.

### **Basics of Energy Management Seminar Outline**

Preface -- Goals and Purpose of This Self-Paced Seminar

**Chapter 1** Introduction to Energy Management

**Chapter 2** What is Energy and National Energy Use

**Chapter 3** Basic Arithmetic and Calculator Skills

**Chapter 4** Basic Algebra and Analytical Skills

**Chapter 5** Energy Units and Energy Conversions

**Chapter 6** Understanding Electrical Systems and Electrical Rates

**Chapter 7** Economic Analysis and Economic Decisions

**Chapter 8** Basic Technologies, Terminology and Units for Equipment End Use Categories - Lighting, Motors, Air Conditioning, and Boilers

### **Order of Operations / Hierarchy:**

The order of operations or hierarchy is the order in which a mathematical expression or equation should be conducted. The hierarchy of operations is as follows:

1. Operations inside grouping symbols
2. Exponents
3. Multiplication and/or Division in the order of left to right
4. Addition and/or subtraction in the order of left to right

Examples:

1.  $7 + 3 \times 4 + 6$

For this expression, the first operation must be the  $3 \times 4$

$$7 + 12 + 6$$

The next order would be the addition, in order from left to right

$$25$$

2.  $6 \times 7^2 \times 5 + 14 \times .9^2$

For this expression, the first operation must be the exponents

$$6 \times 49 \times 5 + 14 \times .81$$

The next order would be the multiplication, in order from left to right

$$1,470 + 11.34$$

The next order would be the addition, in order from left to right

$$1,481.34$$

3.  $133 \times (.06^2 + .125 \times 420) + 12^3$

For this expression, the first operation must be the exponents (we must reduce these to fully do the operation within the grouping symbol)

$$133 \times (.0036 + .125 \times 420) + 1,728$$

The next order would be the multiplication within the grouping symbol

$$133 \times (.0036 + 52.5) + 1,728$$

The next order would be the addition within the grouping symbol

$$133 \times (52.5036) + 1,728$$

The next order would be the multiplication

$$6,982.9788 + 1,728$$

The next order would be the addition

$$\underline{8,710.9788}$$

4.  $1,786 + (7 * 3 \div 3 * 7)^2\{14 + 24 - 14 + 24\} + \left(\frac{80}{100}\right)^3$

For this expression, the first operation must be the operations within the grouping symbols. For the first grouping symbol, multiplication in the order from left to right; the second grouping symbols is addition and subtraction in order from left to right; the third grouping symbol is the performance of the division.

$$1,786 + (49)^2\{48\} + (.80)^3$$

The next order would be the exponents

$$1,786 + 2,401\{48\} + .5120$$

The next order would be the multiplication

$$1,786 + 115,248 + .5120$$

The next order would be addition, in order from left to right

$$\underline{117,034.512}$$

**Important Note: Business calculators are programmed to do the order of operations in the order that you enter them, ie – it ignores hierarchy. Whereas a scientific calculator knows hierarchy and when posed with entering numbers, it will perform the operations following the order of operations above.**

## **Calculators:**

One item that will be of great importance prior to attending the seminar or webinar is purchasing a calculator. There are four main types of calculators: Financial or business, scientific, trade show, and phone applications.

One strong word of warning since we have seen it happen so many times and it creates confusion. DO NOT bring a small trade show calculator or one of the calculators that come embedded in a pad folio notebook – these will not be effective. The reason why they are not effective is in the significant digit storage. Some of the calculations we do will involve very large numbers, they will be in the billions or trillions and may sometimes be in scientific notation. When this happens these smaller trade show type calculators cannot process the entire string of number so it starts scraping off digits which makes the result slightly off because the number is essentially been rounded several times and in the end looks like an error. We personally have seen many attendees get very frustrated with these smaller ones.

Phone application calculators are not allowed. This is because the BEP Board does not allow phones to be out during the exam (this is a consistent policy amongst all of AEE's Certification Boards – mainly to protect the integrity of the exam since most phones now have cameras).

Between financial or business and scientific calculators, we recommend that the scientific calculator be purchased for the BEP Course. Recognize the main difference between these two types of calculators is financial or business calculators do not know hierarchy – i.e. they do not multiplication before addition/subtraction etc for an example, they also do not have exponents or scientific notation. A scientific calculator is all that you will need for the BEP course (as well as all of the other AEE Certification courses. As far as a recommendation, a good calculator is something in the \$15 -20 range, you do not need to go out and buy a \$80plus Texas Instruments (TI)80/81/85 for the class. Something in the TI 30, 30x, 35, 35x range will do just fine. There are many other brands available that will perform quite well such as Casio, etc.

In closing for calculators, give yourself an advantage by getting your calculator NOW, Do not wait until the day of class to take your brand new calculator out of the package. We have seen this many times. Think of this – how would you like to learn a new topic in energy management but you can't because you're still trying to learn the calculator – doesn't work that well. So Please, get your calculator soon and practice with it ☺ – Yes, We did say to practice right !

## Expressions:

An expression is a mathematical phrase that is a combination of one or more numbers, variables, constants combined by one or more operational symbols.

A numerical expression is a mathematical phrase that involves only one or more numbers combined by one or more operational symbols.

Here are five examples of expressions:

$$1,786 + 115,248 + .5120$$

$$140,000 \text{ gal} + 235,000 \text{ gal}$$

$$6X + 4Y + 2Z + 6U$$

$$2.065 \times 10^6 \frac{\text{Btu}}{\text{annually}} + 14.5 \times 10^6 \frac{\text{Btu}}{\text{Annually}}$$

$$\frac{2.065 \times 10^6 \frac{\text{Btu}}{\text{annually}} + 14.5 \times 10^6 \frac{\text{Btu}}{\text{Annually}}}{175,000 \text{ ft}^2}$$

All of the above items are expressions, even the last one which would be how to calculate an energy use index (EUI). They can be reduced or simplified. One key to understanding the difference between an expression and an equation is **there IS NO Equal sign** in an expression. It is not equal to anything, Yes – it can be reduced or simplified or converted to another item but it is not equal to anything.

## BEP Examples of Expressions:

1. Unit conversions, yes – we may for ease of explanation put an equal sign on the end but really it is just been reduced or transformed into another unit.
2. Energy Use Index, again – we may for ease of explanation use an equal sign on the end to put an indicator of what the results of the simplification is, but it is really just being reduced or simplified.

## **Unit Conversions:**

As a Business Energy Professional, it is very important that we can convert from one like energy related unit to another. Since we have several different basic energy units and many different energy unit multipliers, we constantly face the task of converting from one set of energy units to another. These conversions can be handled in a very systematic approach that can be applied to basic conversions, and to more complex conversions and calculations in the future.

The principle of performing unit conversions is simply to carry out algebraically correct multiplications and divisions using correct units at each step, starting with the given piece of information and transforming it into the desired units with the use of one or more conversion factors. If we ever perform one of these basic unit conversion calculations, and find that we have different units on the left and the right. We do not have the correct answer in terms of the desired units. This method is given the colloquial name Railroad Track Method, because the vertical separation lines remind us of railroad tracks. Before we start, here are the conversion factors that we will use in class and because you're in the energy management field – You'll use these extensively in your work!

### **Conversion factors for Unit Conversions:**

1 kWh.....3,412 Btu

1 ft<sup>3</sup> natural gas.....1,000 Btu

1 Ccf natural gas..... 100 ft<sup>3</sup> natural gas

1 Mcf natural gas.....1,000 ft<sup>3</sup> natural gas

1 therm natural gas.....100,000 Btu

1 barrel crude oil.....5,100,000 Btu

1 ton coal.....	25,000,000 Btu
1 gallon gasoline.....	125,000 Btu
1 gallon ethanol.....	84,400 Btu
1 gallon #2 fuel oil.....	140,000 Btu
1 gallon LP gas.....	95,000 Btu
1 HP.....	746 Watts
1 Mbtu.....	1,000 Btu
1 MMBtu.....	1,000,000 Btu
1 Decatherm.....	1,000,000 Btu
1 therm .....	100,000 Btu
1 Quad.....	$1 \times 10^{15}$ Btu
1 MW.....	1,000,000 Watts

Let's walk through a couple of examples:

1. How many quarts are in 10 Gallons of Milk?

(We realize that you probably have the answer already but let's see how to use the process. First draw a common fraction bar (the Track)

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Next we will always write in the top left, what we are given. In this example, we are given 10 gallons of milk

10 Gallons

Next we draw a separation line

10 Gallons |

|

Now we need to use a conversion factor that will convert gallons to quarts, the factor is 4 quarts in 1 gallon. We will need the units of gallons to cancel – so the 1 gallon must go on the bottom

$$\frac{10 \text{ Gallons} \quad | \quad 4 \text{ Quarts}}{| \quad 1 \text{ Gallon}}$$

Next we will cross cancel the units of Gallons, and we're left with

$$\frac{10 \quad | \quad 4 \text{ Quarts}}{| \quad 1}$$

At this point, we simply multiply, 10 times 4 which will equal 40, then bring the units, so in 10 gallons of milk, there are **40 quarts** in 10 gallons of milk.

- Next, let's walk through a little more complex conversion, but again follow the same process. For this example we want to know how many therms are contained in 4,750 gallons of #2 fuel oil?

First draw a common fraction bar (the Track)

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Next we will always write in the top left, what we are given. In this example, we are given 4750 gallons of #2 fuel oil

$$\frac{4,750 \text{ Gallons}}{\hspace{10em}}$$

Next we draw a separation line

$$\frac{4,750 \text{ Gallons} \quad | \quad \hspace{10em}}{| \quad \hspace{10em}}$$

Now we need to use a conversion factor that will convert gallons to therms, but we see from the conversion factor list, there isn't one, so we need to select a middle step by converting the gallons to Btu's, and then converting the Btu's to Therms. In this first step we will need the units of gallons to cancel – so the 1 gallon must go on the bottom

$$\frac{4,750 \text{ Gallons} \quad | \quad 140,000 \text{ Btu}}{| \quad 1 \text{ Gallon}}$$



Next we will cross cancel the units of Gallons, and we're left with

$$\frac{4,750 \quad | \quad 140,000 \text{ Btu}}{|1}$$

Next we will enter the next conversion, from Btu's to Therms

$$\frac{4,750 \quad | \quad 140,000 \text{ Btu} \quad | \quad 1 \text{ Therm}}{|1 \quad | \quad 100,000 \text{ Btu}}$$

Next we will cross cancel the units of Btu's, and we're left with

$$\frac{4,750 \quad | \quad 140,000 \quad | \quad 1 \text{ Therm}}{|1 \quad | \quad 100,000}$$

At this point, we simply multiply and divide; 4,750 times 140,000 and divide this by 100,000. This gives us an answer of **6,650 Therms** of energy in 4,750 gallons of #2 fuel oil.

Now try some examples for yourself. (answer key is on the last sheet)

1. How many kWh are there in 100,000 cubic feet of natural gas?
2. How many Btu's are there in 2.6 Million kWh?
3. How many MMBtu are there in 450,000 therms of natural gas?
4. How many MMBtu are there in 500,000 gallons of #2 fuel oil?
5. How many gallons of gasoline (in Btu content) would be the equivalent to having 5,000 gallons of ethanol?

### **Equations:**

An equation is a mathematical statement in which two expressions are equal to each other. Here are the formal steps to solving equations:

1. Reduce all fractions and combine like terms whenever possible
2. Multiply both side of the equation by the lowest common denominator
3. Isolate all terms that contain the variable to one side of the equation
4. Factor out the variable that you wish to solve for
5. Divide both sides of the equation by the expression the factor came out of
6. Reduce the solution if possible
7. Check your work

Now let's walk through an example:

Solve for X?

$$\frac{24x}{6} = \frac{20x}{100} + 15y - 12z$$

In this first step we will reduce all fractions

$$4x = .2x + 15y - 12z$$

In the next step we will get all of the terms that contain the variable x to one side of the equation. We do this by adding or subtracting. If we move one term to the other side of the equation we need to change its sign. Recognize this is a shortcut, if we showed it the full way in this example – we would show that we would be subtracting .2x from both sides of the equation. On the right side when we do this subtraction it would equal zero)

$$4x - .2x = 15y - 12z$$

In this step we can combine like terms

$$3.8x = 15y - 12z$$

To show how we factor the unknown out we'll factor the variable out

$$x(3.8) = 15y - 12z$$

Then, we'll divide both sides of the equation by the expression in the parenthesis

$$\frac{x(3.8)}{(3.8)} = \frac{15y - 12z}{(3.8)}$$

The 3.8 on the left side will cancel each other out, so the answer to this problem is because it cannot be reduce further

$$x = \frac{15y - 12z}{3.8}$$

Remember the process to solve equations and recognize that the more practice you do the better and faster you will become at solving them. Here are a couple of shortcuts.

Since it's an equation, what you do to one side you have to do to the other to keep it equal. When we look at the final results of this balancing we can move items to the other side of the equation based upon the following:

1. If the term that we are moving is in the numerator we can move it to the other side of equation, we can move it into the numerator by simply changing the sign of it. We can also move it to the other side by moving it to the denominator.
2. If the term that we are moving is in the denominator we can move it to the other side of equation, we can move it to other side if we move it to the numerator.

For the BEP seminar and webinar these are examples of Equations we'll be solving.

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|------------------------|--|
| 1. Point of Use, where | Purchase Price/Efficiency = Point of Use Cost                      |
| 2. Fan Law 1, where    | RPM new / RPM old = CFM new / CFM old                              |
| 3. Fan Law 3, where    | $HP_{new} = HP_{old} \left( \frac{RPM_{new}}{RPM_{old}} \right)^3$ |
| 4. HVAC equations      | $kW/Ton = 12/EER$<br>$kW/Ton = 3.5/COP$<br>$12/EER = 3.5/COP$      |
| 5. Power factor        | $PF = KW/KVA$  |
| 6. Financial           | PV, NPV, AV  |

Let's use a few of these in some examples:

1. On a particular HVAC unit it has a EER of 14. What would the equivalent COP of the unit be?

$$\frac{12}{EER} = \frac{3.5}{COP}$$

$$12COP = 3.5EER$$

$$COP = \frac{3.5EER}{12}$$

$$COP = \frac{3.5 (14)}{12}$$

$$COP = 4.0833$$

2. On the electrical system, the kW delivered is 249,000kW. The power factor of the system is 92%. What is the kVA delivered?

$$PF = \frac{kW}{KVA}$$

$$PF * KVA = kW$$

$$kVA = \frac{kW}{PF}$$

$$kVA = \frac{249,000}{.92}$$

$$kVA = 270,652.17$$

3. Let's try one which involves point of use (POU). What is the POU cost of a natural gas boiler where the purchase price of fuel is \$1.10 a therm. The boiler is currently 73% efficient. (We want the POU to be in units of MMBtu)

$$POU = \frac{\text{Purchase price}}{\text{efficiency}}$$

To make this a little easier, we will do this operation much like a unit conversion.

$$\frac{\$1.10 \quad | \quad 1 \text{ therm} \quad \quad | \quad 1,000,000 \text{ Btu} \quad |}{\text{therm} \quad | \quad 100,000 \text{ Btu} \quad \quad | \quad 1 \text{ MMBtu} \quad \quad | \quad .73}$$

This gives us a POU cost of \$15.07 per MMBtu

Now for the toughest equation solving for the BEP and it's an important concept to get especially for an energy manager who has to deal with fuel switching (ie at what price do you switch to another fuel source). Let's say that a propane boiler vendor is proposing a new boiler that is 92% efficient. Rather than just comparing efficiencies, we also need to consider the cost of the fuel to see if it will really save any money. We need to calculate the point break for propane (this would be the price of propane that would give the same POU cost). As we walk through this problem there are a couple of ways to look at it and we'll walk through both:

First set the POU(gas) = POU(propane)

$$\frac{\$15.07}{MMBtu} = \frac{\$X \mid 1 \text{ gal} \mid 1,000,000 \text{ Btu} \mid}{\text{gal} \mid 140,000 \text{ Btu} \mid 1 \text{ MMBtu} \mid .92}$$

**First method:** to solve for X, we could simply move everything on the right side of the equation to the left, remembering that if it was in the numerator on the right it will go to the denominator on the left, and if it is the denominator on the right, it will go to the numerator on the left.

$$\frac{\$15.07 \mid 140,000 \text{ Btu} \mid 1 \text{ MMBtu} \mid .92}{MMBtu \mid 1 \text{ gal} \mid 1,000,000 \text{ Btu} \mid} = \frac{\$X}{\text{gal}}$$

When we get to this point, you can see that the units of MMBtu and Btu will cancel leaving us the desired units of \$ per gallon. From here it is merely a calculator problem to the answer of \$1.94 per gallon of propane

**Second method:** Some participants prefer to do the algebraic manipulation by moving everything in the right side of the equation underneath the POU gas like this:

$$\frac{\frac{\$15.07}{MMBtu}}{\frac{\mid 140,000 \text{ Btu} \mid 1 \text{ MMBtu} \mid .92}{\mid 1 \text{ gal} \mid 1,000,000 \text{ Btu} \mid}} = \frac{\$X}{\text{gal}}$$

This is OK do to. Just remember to do the operation in the numerator. Then do the operation in the entire denominator. Then divide it out to like this (units not shown for ease)

$$\frac{15.07}{.1288} = \frac{\$X}{\text{gal}}$$

There are more practice problems in the BEP study guide with the two practice quizzes. Please Practice – Did we say Practice – Yes – it will make your seminar and webinar much more meaningful !

**Unit conversion key:**

To help explain these, we have left the units to show the step all as one

1. How many kWh are there in 100,000 cubic feet of natural gas?

$$\frac{100,000 \text{ ft}^3 \quad | \quad 1,000 \text{ Btu} \quad | \quad 1 \text{ kWh}}{| 1 \text{ ft}^3 \quad | \quad 3,412 \text{ Btu}}$$

29,308.32 kWh

2. How many Btu's are there in 2.6 Million kWh?

$$\frac{2,600,000 \text{ kWh} \quad | \quad 3,412 \text{ Btu}}{| 1 \text{ kWh}}$$

8,871,200,000 kWh

3. How many MMBtu are there in 450,000 therms of natural gas?

$$\frac{450,000 \text{ therm} \quad | \quad 100,000 \text{ Btu} \quad | \quad 1 \text{ MMBtu}}{| 1 \text{ therm} \quad | \quad 1,000,000 \text{ Btu}}$$

45,000 MMBtu

4. How many MMBtu are there in 500,000 gallons of #2 fuel oil?

$$\frac{500,000 \text{ gallons} \quad | \quad 140,000 \text{ Btu} \quad | \quad 1 \text{ MMBtu}}{| 1 \text{ gal} \quad | \quad 1,000,000 \text{ Btu}}$$

70,000 MMBtu

5. How many gallons of gasoline (in Btu content) would be the equivalent to having 5,000 gallons of ethanol?

$$\frac{5,000 \text{ gal ethanol} \mid 84,400 \text{ Btu} \quad \mid 1 \text{ gal gasoline}}{\mid 1 \text{ gal ethanol} \quad \mid 125,000 \text{ Btu}}$$

3,376 gallons